AIMS | African Institute for Mathematical Sciences SOUTH AFRICA

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Risk Management and Insurance

Tutorial 3. Compound Interest

As in Tutorial 2, four basic questions can be asked for each method of compounding, with three variables being given and the fourth to be determined. This tutorial includes examples for some (but not all) possible questions.

A) Discrete Time

<u>Question 1</u>

A financial instrument with a face value of \$5000 is due in 4 years and 6 months. What is its value today if the interest rate is 6%, compounded quarterly?

<u>Question 2</u>

Find the time it will take for an investment to double in value at 5%, compounded annually.

<u>Question 3</u>

Find the interest rate, compounded monthly, at which \$2000 will accumulate to \$3000 in 3 years and 9 months.

B) Continuous Time

<u>Question 4</u>

\$10,000 is invested for 4 years at an interest rate of 6%. Find the terminal value if interest is compounded (a) annually and (b) continuously.

<u>Question 5</u>

Find the present value of \$ 20,000 due in 3 years and 2 months if the interest rate is 8%, compounded continuously.

<u>Question 6</u>

What is the interest rate, compounded continuously, at which \$20,000 will accumulate to \$30,000 in five years and 9 months.

<u>Question 7</u>

In how many days will \$10,000 accumulate to \$12,500 with an interest rate of 5%, compounded continuously?

Question 8

a) A bank quotes an interest rate of 6%, compounded continuously. Find the equivalent rate with annual compounding.

b) A credit contract stipulates an interest rate of 8%, compounded annually. Find the equivalent rate with continuous compounding.

Question 9

a) The inflation target of the Reserve Bank of Australia is 2 – 3%. By how much will the purchasing power of the dollar fall over a period of 20 years if the inflation rate is 2.5% per year?

b) How long will it take for the purchasing power of the dollar to fall by one half if the RBA meets its goal and the inflation rate is 2.5% per year?

Tutorial 3

Question 1

Present value:

$$V_0 = \frac{V_t}{\left(1+\frac{R}{m}\right)^{m.t}} = \frac{5000}{\left(1+\frac{0.06}{4}\right)^{4\times\frac{9}{2}}}$$

$$= \frac{5000}{\left(1+\frac{0.06}{4}\right)^{18}} = \underline{\underline{\$\ 3824.56}}$$

Question 2

$$V_t = V_0\,(1+R)^t$$
$$2V_0 = V_0\,(1+R)^t$$
$$2 = (1+R)^t$$
$$\ln 2 = t\,\ln(1+R)$$
$$t = \frac{\ln 2}{\ln(1+R)} = \frac{\ln 2}{\ln(1+0.05)} = 14.2067 \text{ years}$$

$\longrightarrow$ $\underline{\underline{\text{14 years and 75.4 days.}}}$

Tell students that they always should use the natural logarithm and not the logarithm to the base 10 (although it usually does not matter)

Question 3

$$V_t = V_0 \left(1 + \frac{R}{m}\right)^{mt}$$

$$\frac{V_t}{V_0} = \left(1 + \frac{R}{m}\right)^{mt}$$

$$\left(\frac{V_t}{V_0}\right)^{\frac{1}{mt}} = 1 + \frac{R}{m}$$

$$R = \left[\left(\frac{V_t}{V_0}\right)^{\frac{1}{mt}} - 1\right] m$$

$$= \left[\frac{3000}{2000}^{\frac{1}{12 \times 3\frac{9}{12}}} - 1\right] 12 = \left[\frac{3000}{2000}^{\frac{1}{45}} - 1\right] 12$$

$$= 0.1086 \longrightarrow \underline{\underline{10.86\%}}$$

$\longrightarrow$

Question 4

a) $V_4 = 10'000(1+0.06)^4 = \$\ 12'624.77$

b) $V_4 = 10'000\, e^{0.06 \times 4} = \$\ 12'712.49$

It is important to specify the speed of compounding in credit contracts (annually, monthly, daily or continuously)

Question 5

$$V_0 = \frac{V_t}{e^{Rt}} = \frac{20'000}{e^{0.08 \times 3\,1/6}} = \frac{20'000}{e^{0.08 \times 19/6}} = \underline{\underline{\$\ 15'524.18}}$$

Question 6

$$\begin{aligned} V_t &= V_0 \cdot e^{Rt} \\ \ln V_t &= \ln V_0 + Rt \qquad \Big/ \ln e^{Rt} = Rt \\ R &= \frac{\ln V_t - \ln V_0}{t} \\ &= \frac{\ln 30'000 - \ln 20'000}{5\,\frac{3}{4}} = \frac{\ln 30'000 - \ln 20'000}{\frac{23}{4}} \\ &= 0.0705 \longrightarrow \underline{\underline{7.05\,\%}} \end{aligned}$$

Question 7

$$V_t = V_0 \cdot e^{Rt}$$
$$\ln V_t = \ln V_0 + Rt$$
$$t = \frac{\ln V_t - \ln V_0}{R}$$
$$= \frac{\ln 12'500 - \ln 10'000}{0.05} = 4.4629 \text{ years}$$

$$0.4629 \times 365 = 168.9 \text{ days}$$

It takes 4 years and 168.9 days.

Question 8

The two interest rates are equivalent if they produce the same terminal value.

a) $$V_0 (1+R_A)^t = V_0 e^{R_c t}$$
$$1 + R_A = e^{R_c}$$
$$R_A = e^{R_c} - 1$$
$$= e^{0.06} - 1 = 0.06184$$
$$\longrightarrow 6.18\%$$

Since continuous compounding accrues faster than annual compounding, a smaller continuous interest rate produces the same terminal value as a larger annual rate.

b) $$V_0 (1+R_A)^t = V_0 e^{R_C t}$$
$$1 + R_A = e^{R_C}$$
$$\ln (1+R_A) = R_C$$
$$R_C = \ln(1 + 0.08)$$
$$= 0.07696 \longrightarrow 7.70\%$$

Question 9

a)
$$P_{20} = P_0 e^{-rt}$$
$$= P_0 e^{-0.025 \times 20} = 0.6065 P_0$$

The purchasing power of the \$ falls by 39.35 %.

$\longrightarrow$

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b)

$$\underbrace{\frac{1}{2}\,P_0}_{P_t} = P_0\, e^{-\pi t}$$ inflation rate

Cancel P_0 and take logarithm:

$$\ln\frac{1}{2} = -\pi t$$

$$t = -\frac{\ln 0.5}{\pi}$$

$$= -\frac{\ln 0.5}{0.025} = 27.7259$$

It takes 27.7 years for the purchasing power of the $1 to fall by 50%.