Risk Management and Insurance

Tutorial 9. Insurance Supply

Question 1

Suppose that on average 8 patients arrive at the trauma unit of a hospital each day. What is the probability that there will be more than 15 patients on a single day?

The table for the Poisson distribution is attached.

Question 2

Suppose some organization sets up a death benefit fund for its members.

Members	1000
Death benefit (x)	\$ 100,000
Annual mortality rate of a member	0.8%
Equity capital (S_0)	\$ 700,000
Maximum probability of ruin (ϕ)	0.5%
Time horizon	1 year

Determine the safety loading and premium income that is required for the desired maximum probability of ruin of 0.5%.

Question 3

A financial regulator is investigating an insurance company with an equity capital of \$ 200 million. The expected annual indemnities are \$ 60 million with a standard deviation of \$ 20 million and a skewness of 3.6. What is the probability of ruin?

The table for the normal distribution is attached.

Question 4

The finance division of an insurance company has estimated/collected the following data:

Underwriting beta (β_u)	-0.1
Funds generating factor (k)	2
Market return (μ_M)	8%
Risk-free interest rate (r_f)	2%

a) Determine the risk-adjusted expected return on underwriting that is equal to the market return.

b) What is the required premium income if the expected loss is $ 20,000,000?

c) Compare the traditional premium calculation with the premium calculation that is based on the insurance CAPM (not more than three sentences).

Poisson Distribution (cumulative)

Table 6.A.2 Poisson-distribution (cumulative)

n	E(n)														
	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	6.0	7.0	8.0	9.0	10.0
0	0.60653	0.36788	0.22313	0.13534	0.08208	0.04979	0.03020	0.01832	0.01111	0.00674	0.00248	0.00091	0.00034	0.00012	0.00005
1	0.90980	0.73576	0.55783	0.40601	0.28730	0.19915	0.13589	0.09158	0.06110	0.04043	0.01735	0.00730	0.00302	0.00123	0.00050
2	0.98561	0.91970	0.80885	0.67668	0.54381	0.42319	0.32085	0.23810	0.17358	0.12465	0.06197	0.02964	0.01375	0.00623	0.00277
3	0.99825	0.98101	0.93436	0.85712	0.75758	0.64723	0.53663	0.43347	0.34230	0.26503	0.15120	0.08177	0.04238	0.02123	0.01034
4	0.99983	0.99634	0.98142	0.94735	0.89118	0.81526	0.72544	0.62884	0.53210	0.44049	0.28506	0.17299	0.09963	0.05496	0.02925
5	0.99999	0.99941	0.99554	0.98344	0.95798	0.91608	0.85761	0.78513	0.70293	0.61596	0.44568	0.30071	0.19124	0.11569	0.06709
6	1.00000	0.99992	0.99907	0.99547	0.98581	0.96649	0.93471	0.88933	0.83105	0.76218	0.60630	0.44971	0.31337	0.20678	0.13014
7	1.00000	0.99999	0.99983	0.99890	0.99575	0.98810	0.97326	0.94887	0.91341	0.86663	0.74398	0.59871	0.45296	0.32390	0.22022
8	1.00000	1.00000	0.99997	0.99976	0.99886	0.99620	0.99013	0.97864	0.95974	0.93191	0.84724	0.72909	0.59255	0.45565	0.33282
9	1.00000	1.00000	1.00000	0.99995	0.99972	0.99890	0.99669	0.99187	0.98291	0.96817	0.91608	0.83050	0.71662	0.58741	0.45793
10	1.00000	1.00000	1.00000	0.99999	0.99994	0.99971	0.99898	0.99716	0.99333	0.98630	0.95738	0.90148	0.81589	0.70599	0.58304
11	1.00000	1.00000	1.00000	1.00000	0.99999	0.99993	0.99971	0.99908	0.99760	0.99455	0.97991	0.94665	0.88808	0.80301	0.69678
12	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99992	0.99973	0.99919	0.99798	0.99117	0.97300	0.93620	0.87577	0.79156
13	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99992	0.99975	0.99930	0.99637	0.98719	0.96582	0.92615	0.86446
14	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99993	0.99977	0.99860	0.99428	0.98274	0.95853	0.91654
15	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99993	0.99949	0.99759	0.99177	0.97796	0.95126
16	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99998	0.99983	0.99904	0.99628	0.98889	0.97296
17	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99994	0.99964	0.99841	0.99468	0.98572
18	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99987	0.99935	0.99757	0.99281
19	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99996	0.99975	0.99894	0.99655
20	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99991	0.99956	0.99841
21	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99997	0.99983	0.99930
22	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99993	0.99970
23	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998	0.99988
24	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99995
25	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99998

Standard Normal Distribution

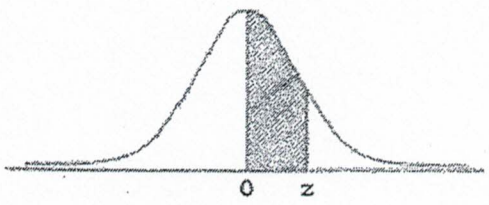

Table 6.A.1 Area below the standard normal density (from 0 up to z)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00000	0.00399	0.00798	0.01197	0.01595	0.01994	0.02392	0.02790	0.03188	0.03586
0.1	0.03983	0.04380	0.04776	0.05172	0.05567	0.05962	0.06356	0.06749	0.07142	0.07535
0.2	0.07926	0.08317	0.08706	0.09095	0.09483	0.09871	0.10257	0.10642	0.11026	0.11409
0.3	0.11791	0.12172	0.12552	0.12930	0.13307	0.13683	0.14058	0.14431	0.14803	0.15173
0.4	0.15542	0.15910	0.16276	0.16640	0.17003	0.17364	0.17724	0.18082	0.18439	0.18793
0.5	0.19146	0.19497	0.19847	0.20194	0.20540	0.20884	0.21226	0.21566	0.21904	0.22240
0.6	0.22575	0.22907	0.23237	0.23565	0.23891	0.24215	0.24537	0.24857	0.25175	0.25490
0.7	0.25804	0.26115	0.26424	0.26730	0.27035	0.27337	0.27637	0.27935	0.28230	0.28524
0.8	0.28814	0.29103	0.29389	0.29673	0.29955	0.30234	0.30511	0.30785	0.31057	0.31327
0.9	0.31594	0.31859	0.32121	0.32381	0.32639	0.32894	0.33147	0.33398	0.33646	0.33891
1.0	0.34134	0.34375	0.34614	0.34849	0.35083	0.35314	0.35543	0.35769	0.35993	0.36214
1.1	0.36433	0.36650	0.36864	0.37076	0.37286	0.37493	0.37698	0.37900	0.38100	0.38298
1.2	0.38493	0.38686	0.38877	0.39065	0.39251	0.39435	0.39617	0.39796	0.39973	0.40147
1.3	0.40320	0.40490	0.40658	0.40824	0.40988	0.41149	0.41308	0.41466	0.41621	0.41774
1.4	0.41924	0.42073	0.42220	0.42364	0.42507	0.42647	0.42785	0.42922	0.43056	0.43189
1.5	0.43319	0.43448	0.43574	0.43699	0.43822	0.43943	0.44062	0.44179	0.44295	0.44408
1.6	0.44520	0.44630	0.44738	0.44845	0.44950	0.45053	0.45154	0.45254	0.45352	0.45449
1.7	0.45543	0.45637	0.45728	0.45818	0.45907	0.45994	0.46080	0.46164	0.46246	0.46327
1.8	0.46407	0.46485	0.46562	0.46638	0.46712	0.46784	0.46856	0.46926	0.46995	0.47062
1.9	0.47128	0.47193	0.47257	0.47320	0.47381	0.47441	0.47500	0.47558	0.47615	0.47670
2.0	0.47725	0.47778	0.47831	0.47882	0.47932	0.47982	0.48030	0.48077	0.48124	0.48169
2.1	0.48214	0.48257	0.48300	0.48341	0.48382	0.48422	0.48461	0.48500	0.48537	0.48574
2.2	0.48610	0.48645	0.48679	0.48713	0.48745	0.48778	0.48809	0.48840	0.48870	0.48899
2.3	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49896	0.49900
3.1	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
3.2	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.3	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
3.4	0.49966	0.49968	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
3.5	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
3.6	0.49984	0.49985	0.49985	0.49986	0.49986	0.49987	0.49987	0.49988	0.49988	0.49989
3.7	0.49989	0.49990	0.49990	0.49990	0.49991	0.49991	0.49992	0.49992	0.49992	0.49992
3.8	0.49993	0.49993	0.49993	0.49994	0.49994	0.49994	0.49994	0.49995	0.49995	0.49995
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997
4.0	0.49997	0.49997	0.49997	0.49997	0.49997	0.49997	0.49998	0.49998	0.49998	0.49998

Tutorial 9

Question 1

Patients arrive at a fixed average rate of 8 patients per day and independently since the last arrival of a patient. Therefore, the Poisson distribution applies.

Using the table, the cumulative probability that there are 15 patients or less is

$$P(X \leq 15) = 0.99177$$

Thus, the probability that there are more than 15 patients is:

$$1 - 0.99177 = 0.00823 \longrightarrow 0.8\%$$

Question 2

Expected number of deaths:

$$E\,n = 0.008 \times 1000 = 8$$

Fair premium income:

$$\Pi = E\,n \cdot x = 8 \times 100.000 = 800.000$$

With an expected value of 8 deaths per year, the cumulative probability that there are up to 16 deaths is:

$$\text{Poisson}\,(16;\,8) = 0.99628$$

- Use the Table or Excel

Thus, the probability that there are more than 16 deaths is

$$1 - 0.99628 = 0.00372 \rightarrow 0.372\,\%$$

From the class notes:

$$u = \frac{S_0 + \bar{\Pi}_7}{X} \qquad (i)$$

The premium income is:

$$\Pi_\theta = (1+\theta)\,\bar{\Pi} = (1+\theta)\,E_u \cdot x$$

Substitute into (i) and solve for θ:

$$u = \frac{S_0 + (1+\theta)\,E_u \cdot x}{x}$$

$$\theta = \frac{ux - S_0}{E_u \cdot x} - 1$$

The loading is:

$$\theta = \frac{16 \cdot 100{,}000 - 700{,}000}{8 \cdot 100{,}000} - 1 = \frac{900{,}000}{800{,}000} - 1$$

$$= 0.125 \longrightarrow 12.5\%$$

Then, the required premium income is:

$$\Pi_\theta = (1+\theta)\,\bar{\Pi} = (1+\theta)\,E_u \cdot x$$

$$= (1+0.125)\,8 \cdot 100{,}000 = \$\,900{,}000$$

With an initial equity capital of \$ 1,500,000 and a premium income of \$ 900,000, the probability of ruin does not exceed 0,5 %.

Question 3

Step 1: Transform the inclinations (loss) into a random variable $\tilde{x}$ with $E(\tilde{x}) = 0$ and $\sigma_{\tilde{x}} = 1$. $\tilde{x}$ is still skewed.

$$\tilde{x} = \frac{S - \mu}{\sigma} = \frac{200 - 60}{20} = 7$$

Step 2: Transform $\tilde{x}$ into the standard normal variable $z \sim \phi(0,1)$.

$$g = \frac{1}{3!} E\tilde{x}^3 = \frac{1}{3!} 3.6 = \frac{3.6}{6} = 0.6$$

$$z = \sqrt{1 + \frac{1}{4g^2} + \frac{\tilde{x}^2}{g}} - \frac{1}{2g}$$

$$= \sqrt{1 + \frac{1}{4 \times 0.6^2} + \frac{7}{0.6}} - \frac{1}{2 \times 0.6} = 2.82$$

Using the standard normal table:

$$P(z > 2.82) = 0.5 - 0.4976 = 0.0024$$

The probability of ruin is 0.24%.

Question 4

a) Use the insurance CAPM:

$$\mu_u = -k r_f + \beta_u (\mu_M - \mu_F)$$

$$= -2 \times 0.02 + 0.1 (0.08 - 0.02)$$

$$= -0.046 \longrightarrow -4.6\%$$

A negative return on underwriting is equivalent to the market return of 8%.

b) Expected return of underwriting:

$$\mu_u \equiv \frac{\bar{\Pi} - L_u}{\bar{\Pi}}$$

- see class notes

It follows:

$$\mu_u \bar{\Pi} = \bar{\Pi} - L_u$$

$$\bar{\Pi} = \frac{L_u}{1 - \mu_u} = \frac{20{,}000{,}000}{1 - (-0.034)} = \$\,19{,}120{,}459$$

-0.046

$\longrightarrow$ Premium income

Loss from underwriting activity:

$$\Pi - L_n = 19{,}342{,}360 - 20{,}000{,}000 = -\$879\,541$$

The profit from investments is important to insurance companies!

c)

The traditional premium calculation uses the loss distribution in a time period to determine the premium income required for a given probability of ruin.

In the insurance CAPM the premium income is given by the condition that the risk-adjusted expected returns of underwriting and investments are equal to the market return. However, it neglects the possibility of ruin.